UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/53**

Paper 5 Mechanics 2 **(M2)** **October/November 2013**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

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UNIVERSITY of CAMBRIDGE
International Examinations

1 A particle P of mass 0.1 kg is attached to one end of a light elastic string of natural length 0.4 m and modulus of elasticity 12 N. The other end of the string is attached to a fixed point O on a smooth horizontal surface. P moves on the surface in a horizontal circle with centre O and radius 0.6 m. Calculate the speed of P. [3]

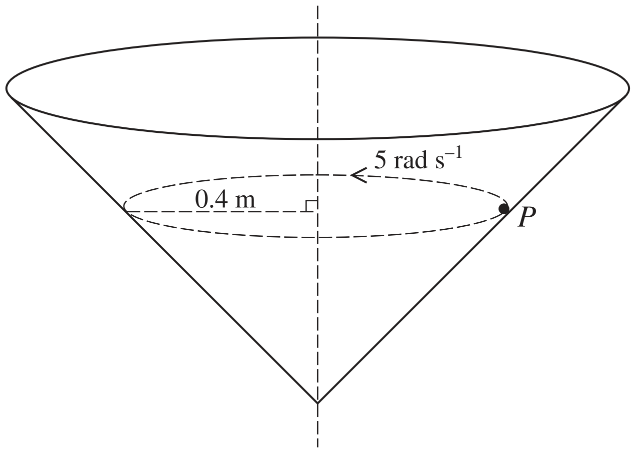
2 A particle P of mass 0.5 kg is released from rest at a point O and falls vertically. When P has downward displacement x m from O, the velocity of P is $v\,\mathrm{m\,s^{-1}}$. A resisting force of magnitude $0.015x^2$ N acts on P.

(i) Show that $v\dfrac{dv}{dx} = 10 - 0.03x^2$. [2]

(ii) Find the value of x when the velocity of P is greatest. [1]

(iii) Calculate the greatest value of v. [4]

3



A particle P of mass 0.5 kg moves in a horizontal circle on the smooth inner surface of a hollow cone which is fixed with its axis vertical and its vertex downwards. P moves with angular speed $5\,\mathrm{rad\,s^{-1}}$ in a circle of radius 0.4 m (see diagram). Show that the semi-vertical angle of the cone is 45° and calculate the magnitude of the force exerted on P by the surface of the cone. [6]

4 A particle P of mass 0.2 kg is projected horizontally with velocity $0.9\,\mathrm{m\,s^{-1}}$ from a point O on a rough horizontal surface. P moves in a straight line, and at time t s after projection the velocity of P is $v\,\mathrm{m\,s^{-1}}$. A force of magnitude $0.024t$ N acts on P in the direction OP. The coefficient of friction between P and the surface is 0.3.

(i) Express the acceleration of P in terms of t, and hence show that, before P comes to rest,

$$v = 0.06(t^2 - 50t + 15).$$ [4]

(ii) Find the value of t when P comes to rest. [2]

(iii) Find the value of t when P subsequently begins to move again. [2]

5 The top of a vertical cliff is 20 m above sea level. A particle P is projected with speed $15\,\mathrm{m\,s^{-1}}$ at an angle of 30° above the horizontal from a point O at the top of the cliff. Calculate

(i) the speed and direction of motion of P when it strikes the water, [4]

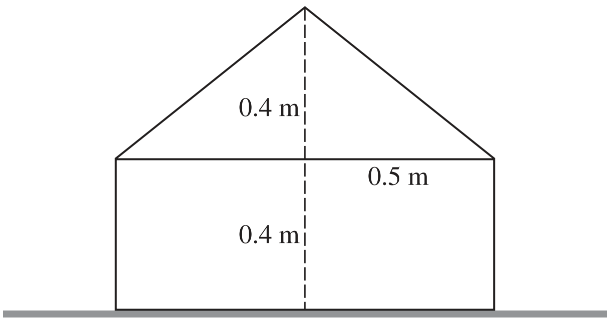
(ii) the distance OP at the instant P strikes the water. [4]

6 One end of a light elastic string of natural length 0.8 m and modulus of elasticity 50 N is attached to a fixed point O. A particle P of mass 0.4 kg is attached to the other end of the string. P is projected downwards with speed $1.5\,\mathrm{m\,s^{-1}}$ from a point 0.82 m vertically below O.

(i) Find the greatest speed of P. [5]

(ii) Show that P cannot reach O. [3]

7



A uniform solid is made from a cylinder and a cone, both with radius 0.5 m and height 0.4 m. The circular base of the cone is attached to a circular face of the cylinder, with their circumferences coinciding. The solid rests in equilibrium with the circular face of the solid on a rough horizontal surface (see diagram).

(i) Show that the centre of mass of the solid is 0.275 m above the surface. [3]

The weight of the solid is 60 N. A horizontal force of increasing magnitude P N is applied to the vertex of the cone which causes the solid eventually to topple without sliding.

(ii) Calculate the value of P for which the solid is on the point of toppling. [2]

(iii) Find the least possible value for the coefficient of friction between the solid and the surface. [1]

The force of magnitude P N is removed, and the solid is held with the curved surface of the cylinder in contact with the horizontal surface. The horizontal surface is then tilted so that it makes an angle of 30° with the horizontal. The solid is released, with its axis of symmetry parallel to a line of greatest slope and the conical portion pointing down the slope.

(iv) Show that the solid does not slide, but does topple. [4]

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